Standard Auslander-Reiten components of a Krull-Schmidt category

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- One introduces Auslander-Reiten quiver $\Gamma_{\mathrm{mod}A}$.
- In general, $\Gamma_{\mathrm{mod}A}$ describes maps not in $\mathrm{rad}^{\infty}(\mathrm{mod}A)$.

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3) (Ringel) Γ is preprojective or preinjective.

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2) *If Γ is regular and standard, then Γ is stable tube or $\Gamma \cong \mathbb{Z}\Delta$, where Δ a finite acyclic quiver.*

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2. In dual situation, f is *sink morphism.*

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REMARK. The above notion unifies almost split sequences in abelian categories and almost split triangles in triangulated categories.

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- *vertices*: the non-isomorphic indecomposables in $\mathcal{A}$.
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- *translation*: if $X \longrightarrow Y \longrightarrow Z$ almost split, then $\tau Z = X$.

Objective

Question

1. How to decide a component of $\Gamma_{\mathcal{A}}$ is standard?

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1. How to decide a component of $\Gamma_{\mathcal{A}}$ is standard?
2. Are there new types of standard components?
3. We consider these problems for components with a section.

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2. Δ meets each τ-orbit in Γ exactly once,
3. Δ is convex in Γ.

Example

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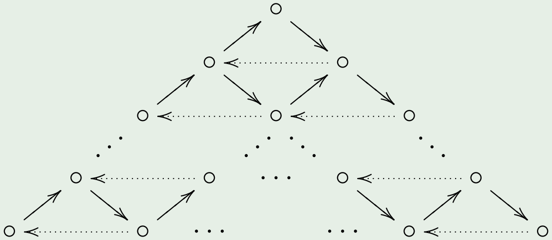

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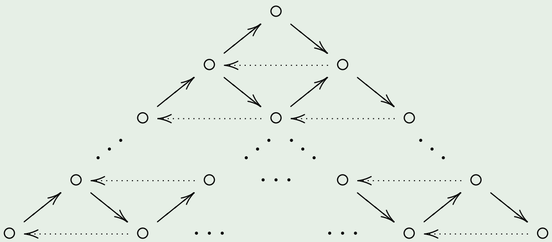

The two longest paths are sections.

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Notation

- $\mathbb{N}\Delta =< (x, i) \mid x \in \Delta_0, i \in \mathbb{N} > \subseteq \mathbb{Z}\Delta$.

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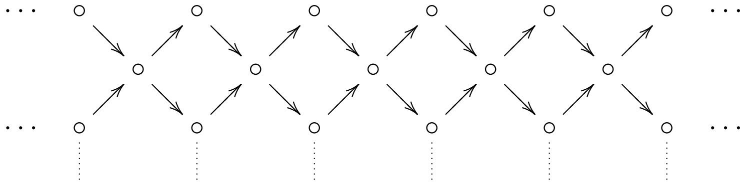
- $\mathbb{N}^{-}\Delta = < (x, -i) \mid x \in \Delta_0, i \in \mathbb{N} > \subseteq \mathbb{Z}\Delta$.

Example

The translation quiver $\mathbb{Z}\mathbb{A}_\infty$ is as follows:



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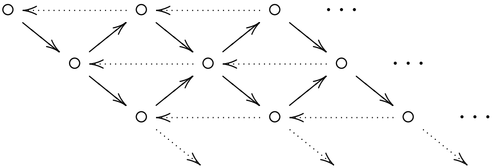

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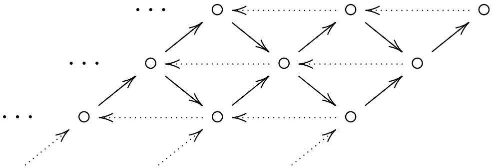
then $\mathbb{N}^-\mathbb{A}_\infty^-$ is as follows:



Properties of components with sections

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Theorem

If Γ is stable, then Γ is τ-periodic or $\Gamma \cong \mathbb{Z}\Delta$ with Δ acyclic quiver.

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Standardness for quasi-serial components

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$\mathrm{proj}(Q)$: additive category of the P_x, $x \in Q_0$.

Finitely presented representations

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where $P_0, P_1 \in \mathrm{proj}(Q)$.

Category of finitely presented representations

$\text{rep}^+(Q)$: finitely presented representations of Q.

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Proposition

$\mathrm{rep}^+(Q)$ *is Hom-finite, hereditary, abelian.*

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$\Delta^- = \emptyset$ and Δ^+ no left-∞ path.

$\mathrm{add}(\Delta) \cong kQ^{\mathrm{op}}$ and $\mathrm{Hom}(\Delta^+, \Delta) = 0$.

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Regular components

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2. *The regular components are all standard $\Leftrightarrow$ Q of infinite Dynkin types $\mathbb{A}_\infty, \mathbb{A}_\infty^\infty, \mathbb{D}_\infty$.*

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1. $\Gamma_{\mathrm{rep}^+(Q)}$ *has at most four components, at most two regular, all standard.*

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2. *Wings,* $\mathbb{Z}\mathbb{A}_\infty, \mathbb{N}\mathbb{A}_\infty^+, \mathbb{N}^-\mathbb{A}_\infty^-$ *all appear in this setting.*

The derived category $D^b(\text{rep}^+(Q))$

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2. $D^b(\mathrm{rep}^+(Q))$ is Hom-finite, Krull-Schmidt.
3. $\Gamma_{D^b(\mathrm{rep}^+(Q))}$ has a *connecting component* $\mathcal{C}_Q$, containing
   - the preprojective component of $\Gamma_{\mathrm{rep}^+(Q)}$.
   - shift by -1 of all preinjective components of $\Gamma_{\mathrm{rep}^+(Q)}$.

Standard components in $D^b(\mathrm{rep}^+(Q))$

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3. *Q of infinite Dynkin type $\Rightarrow \Gamma_{D^b(\mathrm{rep}^+(Q))}$ has at most 3 components up to shift, all standard.*

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2. *Γ is standard with a section $\Leftrightarrow$ Γ is a connecting component of AR-quiver of a tilted factor algebra of A.*