Covering theory for linear categories with application to derived categories

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3. Asashiba has worked along this direction to the level of categories of perfect complexes.

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A functor $F : \mathcal{A} \to \mathcal{B}$ between k-categories is called a *covering* provided, for any $M \in \mathcal{A}$ and $N \in \mathcal{B}$, that F induces two isomorphisms

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Remark

This notion works well only if $\mathcal{A}$ is skeletal (that is, non-zero objects are indecomposable and distinct objects are not isomorphic).

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Lemma

Let G be torsion-free. If the G-action on $\mathcal{A}$ is locally bounded, then it is free.

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and
$$\begin{array}{rcl} \bigoplus_{g\in G} \mathcal{A}(g\cdot X, Y) & \to & \mathcal{B}(F(X), F(Y)) \\ (v_g)_{g\in G} & \mapsto & \sum_{g\in G} F(v_g) \circ \delta^{-1}_{g,X}. \end{array}$$

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Remark

If $\mathcal{A}, \mathcal{B}$ are skeletal, then Galois covering $F : \mathcal{A} \to \mathcal{B}$ in Gabriel's sense is Galois G-covering with trivial G-stabilizer.

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That is, G-precovering is Galois G-covering $\Leftrightarrow$ it is dense.

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Proposition

If $\mathcal{A}$ is triangulated, then a sequence $X \xrightarrow{u} Y \xrightarrow{v} Z$ with $Y \neq 0$ is AR-sequence $\Leftrightarrow$ it embeds in AR-triangle

$$X \xrightarrow{u} Y \xrightarrow{v} Z \xrightarrow{w} X[1].$$

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Corollary

$\mathcal{A}$ has AR-sequences $\Leftrightarrow$ $\mathcal{B}$ has AR-sequences.

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- ϕ induces, for each $x \in \Gamma_0$, two bijections

$$x^+ \to (\varphi(x))^+; \qquad x^- \to (\varphi(x))^-.$$

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- if $x \in \Delta_0$ and $b \in (\varphi(x))^-$, then $v'_{b,\varphi(x)} = \sum_{z \in x^- \cap \varphi^-(b)} u'_{z,x}$.

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- *$\Gamma \cong F(\Gamma)$ in case G_{Γ} is trivial.*

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4. The canonical projection $\pi : D^b(\mathcal{H}) \to \mathscr{C}_{\mathcal{H}}$ is a Galois G-covering.
5. The components of $\Gamma_{\mathscr{C}_{\mathcal{H}}}$ are $\pi(\Gamma)$, where Γ components of $\Gamma_{D^b(\mathcal{H})}$ containing objects in $\mathcal{H}$.

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4. The canonical projection $\pi : D^b(\mathcal{H}) \to \mathscr{C}_{\mathcal{H}}$ is a Galois G-covering.
5. The components of $\Gamma_{\mathscr{C}_{\mathcal{H}}}$ are $\pi(\Gamma)$, where Γ components of $\Gamma_{D^b(\mathcal{H})}$ containing objects in $\mathcal{H}$.
6. If Γ is such a component of $\Gamma_{D^b(\mathcal{H})}$, then $\pi(\Gamma) \cong \Gamma \Leftrightarrow \Gamma$ contains no projective object or no injective object of $\mathcal{H}$.

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2. *If $M, N \in \mathrm{mod}\widetilde{\Lambda}$, then*
$$\oplus_{g\in G} \mathrm{Hom}_{\widetilde{\Lambda}}(M, g \cdot N) \cong \mathrm{Hom}_{\widetilde{\Lambda}}(M, \oplus_{g\in G}\, g \cdot N).$$

Results on module categories by Bongartz-Gabriel

$\pi : \widetilde{\Lambda} \to \Lambda$ induces push-down functor and pull-up functor:

$$\pi_\lambda : \mathrm{Mod}\widetilde{\Lambda} \to \mathrm{Mod}\Lambda; \quad \pi_\mu : \mathrm{Mod}\Lambda \to \mathrm{Mod}\widetilde{\Lambda}.$$

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4. If Λ is locally representation finite, then π_λ induces Galois covering between AR-quivers $\pi_\lambda : \Gamma_{\mathrm{mod}\,\widetilde{\Lambda}} \to \Gamma_{\mathrm{mod}\,\Lambda}$.

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Definition

1) If $w = \alpha_r^{d_r} \cdots \alpha_1^{d_1}$ is a walk in Q, where $d_i = \pm 1$ and $\alpha_i \in Q_1$, then its *degree* is $\partial(w) = d_1 + \cdots + d_r$.

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Example

Any tree is gradable.

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5. The arrows in Q are of the form $x \to y$, where $x \in Q^{(n)}$ and $y \in Q^{(n+1)}$ for some n.

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Theorem

If Q is gradable, then there exists a triangle-equivalence:

$$\mathcal{E} : D^b(\mathrm{rep}^-(Q^{\mathrm{op}})) \to D^b(\mathrm{mod}\, A),$$

$\mathrm{rep}^-(Q^{\mathrm{op}})$: *finitely co-presented representations of Q^{op}.*

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- *In particular,* G *is torsion-free, acting freely on* $\widetilde{Q}$.

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Proposition

The quiver-covering $\pi : \widetilde{Q} \to Q$ induces Galois G-covering of locally bounded k-categories with trivial G-stabilizer

$$\pi : \widetilde{A} \to A.$$

Key Property

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Lemma

The induced push-down functor

$$\pi_\lambda^C : RC^{-,b}(\operatorname{proj} \widetilde{A}) \to RC^{-,b}(\operatorname{proj} A)$$

is dense.

Main Result

Theorem

The Galois G-covering $\pi : \widetilde{A} \to A$ *induces Galois G-covering*

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The Galois G-covering $\pi : \widetilde{A} \to A$ *induces Galois G-covering*

$$\pi_\lambda^D : D^b(\mathrm{mod}\widetilde{A}) \to D^b(\mathrm{mod}\, A).$$

- *If* Γ *is component of* $\Gamma_{D^b(\mathrm{mod}\widetilde{A})}$*, then* $\rho \cdot \Gamma = \Gamma[r_Q]$*, hence,* $\pi_\lambda^D(\Gamma) \cong \Gamma$*.*

- *Proof.* For $X^\cdot \in D^b(\mathrm{mod}\, A)$, $\exists P^\cdot \in RC^{-,b}(\mathrm{proj}\, A)$ such that $X^\cdot \cong P^\cdot$ in $D(\mathrm{mod}\, A)$.

Main consequence

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in such a way that

1. *if* $r_Q = 0$*, then* $\mathcal{F}$ *is equivalence;*
2. *otherwise,* $\Gamma_{D^b(\mathrm{mod}\, A)}$ *consists of* r_Q *copies of the fundamental domain of* $\Gamma_{D^b(\mathrm{rep}^-(\widetilde{Q}\,))}$.

Shapes of AR-components

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- *and some possible stable tubes (this occurs only if Q is Euclidean).*

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6. $K^b(\mathrm{proj} A)$ is symmetric or 0-CY, that is,
   $$D\mathrm{Hom}(X^\cdot, Y^\cdot) \cong \mathrm{Hom}(Y^\cdot, X^\cdot).$$